XX. *M.* De Luc's *Rule for measuring Heights by the Barometer, reduced to the* English *Measure of Length, and adapted to* Fahrenheit's *Thermometer, and other Scales of Heat, and reduced to a more convenient Expression.* By *the Astronomer Royal.*

Redde, Jan. 13, 1774.

M. DE LUC, F. R. S. in a large and valuable Treatise upon the Barometer and Thermometer, lately published at Geneva, the result of many years labour and study, has given a rule for the measurement of heights by the barometer, deduced from his experiments, and far more accurate, than any published before; since it appears that he could determine heights by it generally to 10 or 15 feet, and that the error seldom, if ever, amounted to double that quantity. This valuable degree of exactness he has obtained principally by detecting the faults of the common barometer, and, in consequence, improving the construction of it; and by introducing the use of the mercurial thermometer, to accompany that of the barometer. The principal faults, which he found in the common barometers, arose from the

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the repulfion of the quickfilver by the glafs tube, from air and moifture admitted into the tube, and from the variations of the denfity of quickfilver by heat and cold; another very confiderable error arofe, in calculating heights from the barometer, by not allowing for the changes of the denfity of the air, whofe gravity affords us this meafure of heights, owing to heat and cold. The firft caufe of error, that of the repulfion of the tubes, he remedied, by fubftituting a fyphon-barometer inftead of the fimple upright tube, the repulfion of the two legs of the fyphon, counteracting itfelf; the error arifing from air and moifture in the tube he cured by boiling the quickfilver, after it was put into the tube, and other precautions; the errors, in the eftimation of the heights, arifing from the changes of the denfity of the quickfilver, and denfity of the air by heat and cold, he fhews how to correct by allowances depending on two thermometers, one attached to the frame of the barometer itfelf, and the other made to be expofed to the open air, to fhew its degree of heat; which thermometers are to be noted both at the top and bottom of the hill. Laftly, by a great number of experiments made with accurate barometers and thermometers of h s own conftruction, he has deduced a rule for calculating heights of places; the exactnefs of which he has fufficiently proved by a large table of experiments. But this rule is exprefed in French meafure, and is adapted either to a thermometer, whofe freezing point is o, and that of boiling water 80, or to thermometers of particular fcales. It

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may be therefore uſeful, to reduce M. DE LUC's rule to Engliſh meaſure, and to adapt it to the thermometer of Fahrenheit's ſcale, which is generally uſed in this country.

M. DE LUC, in the winter ſeaſon, heated the air of his room to as great degree as he could, and noted the riſe of the barometer, owing to the diminution of its denſity, or ſpecific gravity, by heat; he alſo noted the height of the thermometer, both before and after the room was heated. Hence he deduced a rule, that when the barometer is at 27 French inches, which was the caſe in this experiment, an increaſe of heat, from freezing to that of boiling water, will raiſe the barometer 6 lines, or $\frac{1}{54}$th part of the whole. It is eaſy to ſee, that when the barometer is higher than 27 French inches, this variation will increaſe in the ſame proportion; or will be always $\frac{1}{54}$th of the heighth of the barometer; therefore, if the height of the barometer be called B, the riſe of the barometer, for an increaſe of heat from freezing to boiling water, will be $\frac{B}{54}$; and, as it will be leſs for a leſs difference of heat, therefore, if the number of degrees, marked on the thermometer, between freezing and boiling water, be called K, and the riſe of the thermometer from any given point be called H, the correſpondent riſe of the barometer will be $\frac{B}{54} \times \frac{H}{K}$, by the increaſe of heat from the given point by the number of degrees H. If the heat, inſtead of increaſing, was to decreaſe, then H would

would fignify fo many degrees decreafe of heat, and the barometer would fink by $\frac{B}{54} \times \frac{H}{K}$. The fixt temperature of heat, to which. M. DE LUC thought beft to reduce his obfervations of the barometer, is $\frac{1}{8}$th of the interval from freezing to boiling water above the former point: and if the thermometer was higher than this degree, he fubftracted $\frac{B}{54} \times \frac{H}{K}$; if it was lower, he added it to the obferved height of the barometer; and thus he obtained the exact height of the barometer, fuch as it would have been, i⌐ ₋₋ denfity of its quickfilver had been the fame as anfwers to the fixt degree of temperature. He thus corrected the height of both his barometers (that at the bottom, and that at the top of the hill) for the particular degree of heat, indicated by a thermometer attached to the barometer, at each ftation; for it might and would commonly happen, that the degree of heat would be different at the two ftations. The heights of the barometers, thus corrected, were what he made ufe of in his fubfequent calculations. Calling thefe two altitudes of the barometer B and b, putting log. B and log. b, for the logarithms of B, and b, taking only the four firft places of figures, after the characteriftic, or confidering the remaining figures as decimals, and putting C for the mean height of a thermometer, expofed to the air at top and bottom of the hill, the freezing point being 0, and the point of boiling water at 80, he finds, by his experiments, that the height of the hill will be given in French toifes, when C is $16\frac{3}{4}$, by

fimply taking the difference of the logarithms of the heights of the barometer, or will be equal to log. B — log. b ; and in any other degree of heat, will be greater or lefs, in proportion as the rarity of the air is greater or lefs, than in the fixt temperature ; or greater or lefs by $\frac{1}{215}$th part of the whole, for every degree of the thermometer reckoned from the fixt temperature $16\frac{3}{4}$; and confequently the height of the place will be expreffed generally in French toifes, by this *formula*

$$\text{log. } B - \text{log. } b + \overline{\text{log. } B - \text{log. } b} \times \frac{C - 16\frac{3}{4}}{215}$$

$$= \overline{\text{log. } B - \text{log. } b} \times 1 + \frac{C - 16\frac{3}{4}}{215}.$$ To reduce this

formula to Englifh meafure, and to the fcale of Fahrenheit's thermometer, we fhould firft premife fome particulars. The French foot is to the Englifh foot, as 1,06575 to 1, as was found by a very accurate experiment: fee Phil. Tranf. Vol. LVIII. for 1768, p. 326; and it is well known, that the point of freezing, on Fahrenheit's thermometer, is at 32, and that of boiling water at 212, or the interval between them 180 degrees. But M. DE LUC's point of boiling water 80, was marked when the barometer was at 27 French inches; and it is the cuftom of our principal Englifh workmen to mark the point of boiling water, 212, on Fahrenheit's thermometer, when the barometer ftands at 30 inches, which is equal to 28 inches, 1,8 lines French meafure; or 13,8 lines higher than M. DE LUC's barometer, when he fet off the point of boiling water on his thermometers ; and it is well known, that the heat of boiling water varies with the weight of the atmofphere: M. DE LUC finds, by his experiments,

ments, this rule, that an increafe of 1 line in the
height of the barometer, raifes the quickfilver
of the thermometer, placed in boiling water, by
$\frac{1}{1134}$th part of the interval between the freezing
point and that of boiling water: he afterwards
indeed found, that this rule would not anfwer for
fuch large variations of the barometer, as take place
in afcending to very great heights above the earth's
furface; *vid. Effai fur les variations du chal: de
l'eau bouill:* but it is accurate enough for any
fmall variation of the barometer, on one fide or
other of its mean height in thefe lowelt regions of
the atmofphere. The change therefore of the boiling
point on Fahrenheit's fcale, for a change of 1 line
in the barometer, will be $\frac{180}{1134} = 0,16$; therefore
13,8 lines will caufe $0,16 \times 13,8 = 2,2$ degrees of
Fahrenheit's fcale; and a thermometer, whofe point
of boiling water was marked 212, when the ba-
rometer ftood at 30 Englifh inches=28 inches,
1,8 lines French meafure, will, when the barome-
ter defcends to 27 French inches, fink 2,2 degrees
in boiling water, or to 209,8, or in round num-
bers to 210 degrees, which is diftant only 178
from 32, the point of freezing. Hence an ex-
tent of 80° of M. DE LUC's thermometer anfwers
to an extent of 178 of our Fahrenheit's thermome-
ter; and putting F for the degrees of this thermo-
meter, correfponding to C of M. DE LUC's, we fhall
have C : F — 32 :: 80 : 178, and $C = \overline{F - 32} \times \frac{80}{178}$
which fubftituted in M. DE LUC's *formula,* gives

$$\overline{\log. B - \log. b} \times 1 + \frac{C - 16\frac{1}{4}}{215} =$$

$$= \overline{\log. B - \log. b} \times 1 + \frac{\overline{F - 32 \times \frac{80}{178}} - 16\frac{1}{4}}{215} =$$

$$= \overline{\log. B - \log. b} \times 1 + \frac{80}{178 \times 215} \overline{F - 32 - \tfrac{17\frac{1}{2}}{10}} \; 16,75$$

$$= \overline{\log. B - \log. b} \times 1 + \frac{F - 32 - 37,27}{478,38} =$$

$$= \overline{\log. B - \log. b} \times 1 + \frac{F - 69,27}{478,38}.$$ Where the

anfwer will ftill come out in French toifes, though adapted to Fahrenheit's thermometer. To bring it out in Englifh fathom (or meafure of 6 feet) multiply the above expreffion by $1,06575$, and we fhall

have $\overline{\log. B - \log. b} \times 1 + \frac{F - 69.27}{478,38} \times 1,06575$

$$= \overline{\log. B - \log. b} \times \overline{409,11 + F} \times \frac{1,06575}{478,38} =$$

$$= \overline{\log. B - \log. b} \times \frac{409,11 + F}{448,87}, \text{ or in round numbers}$$

$$= \overline{\log. B - \log. b} \times \frac{\overline{409 + F}}{449} =$$

$$= \overline{\log. B - \log. b} \times 1 + \frac{F - 40}{449};$$ which will

exprefs the height between the two ftations in Englifh fathom.

In the foregoing expreffions, B and b, as has been mentioned before, fignify heights of the barometer, at the lower and higher ftations, both corrected according to Mr. DE LUC's directions, for the difference of heat between a fixed temperature, (namely $\frac{1}{4}$th of the interval between freezing and boiling water), and the prefent heat, indicated by the thermometer attached to the barometer at each ftation ; but it is not neceffary, to correct *both* barometers for the effect of heat, but only one for the difference of heat of the two; which will be more convenient alfo on another account, becaufe the difference of heat, at the two ftations, will

will be generally fmall, and the correction to re-
duce one barometer to the heat of the other will
confequently be fmall alfo; whereas the difference
of the prefent heat, and the fixt temperature, and
confequently the correction of both barometers,
may be frequently very confiderable: this is
evident : becaufe if the heat of the barometers, at
both ftations, was the fame, however different from
the fixt temperature chofen by M. DE LUC, no
correction would be neceffary; the mercury in the
barometer in both ftations, being expanded in the
fame proportion, and confequently the difference
of the logarithms of its height, at both ftations,
being the fame, as if the heat of both barometers had
agreed with that of the fixt temperature. I fhall now
therefore fuppofe the upper barometer is to be
corrected, to reduce it to the temperature of the
lower one, and that b fignifies the height of this ba-
rometer, as obferved, and not yet corrected; the
correction, from what has been faid above, calling D
the difference of height of the thermometer attach-
ed to the barometer at the two ftations, will be

$\pm \dfrac{D\,b}{54\,K}$, according as the thermometer ftands

higheft at the lower or upper ftation; and the
upper barometer corrected, inftead of b, will be

$b \pm \dfrac{D\,b}{54\,K}$, which fubftituted in the *formula*, gives

$\log. B - \log. \left(b \pm \dfrac{D\,b}{54\,K}\right) \times 1 + \dfrac{F-40}{449}$. But the cor-

rection, on account of the difference of heat of the
barometer at the two ftations, may be reduced to
a ftill eafier expreffion, in which the variable
quantity b, the height of the upper barometer,

fhall

fhall not appear. The fluxion of a logarithm is to
the fluxion of its natural number, as the modulus of
the fyftem to the natural number; and 4343 is the
modulus of the common logarithms, when the four
places, next following the characteriftic, are taken as
whole numbers, inftead of decimals, which is meant
to be done in the ufe of the foregoing *formula*.
Therefore $\frac{Db}{54\,K}$ being very fmall with refpect to b,
we fhall have, variation of log. b : variation of b
$\left(= \frac{Db}{54\,K}\right)$ :: 4343 : b very nearly, and thence va-
riation of log. $b = \pm \frac{Db}{54\,K} \times \frac{4343}{b} = \pm \frac{4343\,D}{54\,K}$.
Which (putting $K = 178$) $= \pm 0,452\,D$. Hence
log. $\left(b \pm \frac{Db}{54\,K}\right) = $ log. $b \pm 0,452\,D$; which
being fubftituted in the *formula* above, will give
the difference of height, of the two ftations, in
Englifh fathom, in a more convenient expreffion,
namely log. $\overline{B -$ log. $b \mp 0,452\,D} \times \overline{1 + \frac{F - 40}{449}}$;
where the upper fign, —, is to be ufed, when the
thermometer of the barometer is higheft at the
lower ftation, and the lower fign, $+$, is to be ufed,
when the faid thermometer is loweft at the lower
ftation. The firft cafe will be moft common; efpe-
cially where the difference of height of the two
ftations is confiderable. It fhould alfo be obferved,
that when F, the height of Fahrenheit's thermo-
meter, is lefs than 40°, $+ \frac{F-40}{449}$ becoming nega-
tive or fubftractive, muft be applied in the calcu-
lation accordingly.

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It may perhaps be convenient to repeat here the meaning of the algebraic terms, ufed in the fore-going *formula*, that any perfon may make ufe of it, without having occafion to recur to the foregoing inveftigation. B fignifies the obferved altitude of the barometer at the lower ftation, and *b* that at the upper ftation; Log. B and Log. *b*, fignify their logarithms taken out of the common tables, by affuming the four firft figures next following the characteriftic as whole numbers, and confidering the three remaining figures, to the right hand, as deci-mals; D fignifies the difference of height of Fahren-heit's thermometer, attached to the barometer at the top and bottom of the hill; and F fignifies the mean of the two heights of Fahrenheit's thermome-ter, expofed freely for a few minutes to the open air in the fhade, at the top and bottom of the hill.

The *formula*, for the meafure of heights, may alfo be changed, and adapted to thermometers of particular fcales, for the convenience of calcula-tion, as M. DE LUC has done; but thefe fcales will be different from his. The thermometer, at-tached to the barometer, had better be divided with the interval between freezing and boiling water, confifting of 81,4 degrees ($=180 \times , 452$) the freezing point may be marked 0, and the point of boiling water will be 81,4; for then, if the difference of height of this thermometer, at the two ftations, be called *d*, we fhall have $d=,$ 0,452 $\times$ D. for $d : D :: 81,4 : 180 :: 0,452 : 1$, and the number of degrees expreft by *d*, will fhew immediately the correction for the difference of heat of the two barometers. If the thermometer, defigned to fhew the temperature of the air, be

divided

divided with the interval between freezing and boiling water $= 200$, and the freezing point be marked $- 9$, and the boiling point $+ 191$, and the heights of this thermometer, at the two ftations, be called G and I, we fhall have $\dfrac{F - 40}{449} = \dfrac{G + I}{2 \times 500}$ $= \dfrac{G + I}{1000}$. For F—40=F—32—8, is the height of Fahrenheit's thermometer, reckoned from 8 degrees above freezing, and $449 : 500 :: 180 : 200 :: 8 : 9$, and the fraction $\dfrac{F - 32 - 8}{449}$, if both the numerator and denominator be increafed in the ratio of 449 to 500, will become $= \dfrac{F - 32 - 8 \times \frac{500}{449}}{500}$ $= \dfrac{F - 32 \frac{500}{449} - 9}{500} = \dfrac{G + I}{2 \times 500} = \dfrac{G+I}{1000}$, becaufe $\dfrac{G+I}{2} + 9$ $= F - 32 \times \frac{500}{449}$. Therefore, if the thermometer of the barometer has the freezing point marked 0, and the point of boiling water 81,4, and the difference of its height, at the two ftations, be called d; and the thermometer for meafuring the temperature of the air, be divided with the interval of 200 between the freezing point and that of boiling water, and the firft be marked $- 9$, and the latter $+ 191$, and the degrees, fhewn by this, at the two ftations, be called G and I; the *formula*, that will give the height of the upper ftation above the lower one, in Englifh fathoms, will be $\overline{\log. B} - \log. \overline{b \mp d} \times 1 + \dfrac{G + I}{1000}$; which confequently multiplied by 6, will give the height in Englifh feet. It is to be obferved, as before, that $- d$ or $+ d$ is to be ufed, according as the thermometer, attached to the barometer, is higheft

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at the lower or upper ftation; and if G and I fhould happen to fall below o of the fcale, or to be fubftractive, they muft be applied accordingly in the calculation.

I fhall now add nothing more, but to give the rule for finding heights by the barometer, according to the *formulæ* delivered above, in common language; firft, as adapted to Fahrenheit's thermometer, and next, as adapted to the two thermometers of particular fcales. Take the difference of the tabular logarithms of the obferved heights of the barometer, at the two ftations, confidering the 4 firft figures, exclufive of the index, as whole numbers, and the three remaining figures to the right as decimals, and fubftract or add $\frac{452}{10000}$th of the difference of the altitude of the Fahrenheit's thermometer, attached to the barometer at the two ftations, according as it was higheft at the lower or upper ftation; thus you will have the height of the upper ftation above the lower, in Englifh fathom, nearly; to be corrected, as follows: make this proportion; as 449 is to the difference of the mean altitude of Fahrenheit's thermometer, expofed to the air at the two ftations, from 40°, fo is the height of the upper ftation found nearly, to the correction of the fame; which added or fubftracted, according as the mean altitude of Fahrenheit's thermometer was higher or lower than 40°, will give the true height of the upper ftation above the lower, in Englifh fathoms; and multiplied by 6, will give it in Englifh feet.

The fame rule, adapted to the thermometers of particular fcales, is this:

Take the difference of the tabular logarithms of the obferved heights of the barometer, at the two ftations, confidering the 4 firft figures (exclufive of the index) as whole numbers, and the three remaining figures to the right as decimals; and fubftract or add the difference of the thermometer, of a particular fcale, attached to the barometer, at the two ftations, according as it was higheft at the lower or upper ftation, and you will have the height of the upper ftation above the lower one, in Englifh fathom, nearly; to be corrected as follows : make this proportion; as 1000 is to the fum of the altitudes of the thermometer of a particular fcale, expofed to the air at both ftations, fo is the height of the upper ftation above the lower, found nearly, to the correction of the fame; which added or fubftracted, according as the fum of the altitudes of the thermometers, expofed to the air, is pofitive or negative, will give the true height of the upper ftation above the lower in Englifh fathoms; and multiplied by 6, will give it in Englifh feet.

NEVIL MASKELYNE.